

\* One fct must be a polynomial

\* Products that one function is not the derivative of the other

Chapter 6: Differential Equations

6.3: Tabular Integration

What you'll Learn About

- How to integrate a product by that cannot be done by recognition

Proof of Integration by Parts

1. Find  $\frac{d}{dx}(uv) =$

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

2. Integrate both sides

$$\int \frac{d}{dx}(uv) = \int u \frac{dv}{dx} + \int v \frac{du}{dx}$$

$$uv = \int u \frac{dv}{dx} + \int v \frac{du}{dx}$$

3. Solve for  $\int u \frac{dv}{dx}$

$$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$$

$$\int u dv = uv - \int v du$$

$$\int x e^x$$

① Derivative of left side

② Antiderivative of right side

Use ultra violet minus super vdu to integrate the following

$$2. \int x e^x dx = x e^x - \int 1 e^x dx = x e^x - e^x + C$$

$$\int u dv = uv - \int v du$$

$$u = x \quad dv = e^x dx$$

$$\frac{du}{dx} = 1 \quad v = e^x$$

$$du = 1 dx$$

Use tabular integration to integrate the following

$$2. \int x e^x dx = x e^x - \int 1 e^x dx = x e^x - e^x + C$$

$$\int \begin{array}{l} x \\ \hline 1 \end{array} e^x dx$$

$$\int \begin{array}{l} x \\ \hline 1 \\ \hline 0 \end{array} e^x$$

Use tabular integration to integrate the following

$$\int x^2 e^{-x} dx =$$

2x	-e <sup>-x</sup>
2	e <sup>-x</sup>
0	-e <sup>-x</sup>

$$6. \int x^2 e^{-x} = -x^2 e^{-x} - \int -2x e^{-x} dx$$

$$= -x^2 e^{-x} + \int 2x e^{-x} dx$$

$$= -x^2 e^{-x} + [-2x e^{-x} - \int -2e^{-x} dx]$$

$$= -x^2 e^{-x} - 2x e^{-x} + \int 2e^{-x} dx$$

$$= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C$$

$$8. \int x^2 \cos\left(\frac{x}{2}\right) = 2x^2 \sin\left(\frac{x}{2}\right) + 8x \cos\left(\frac{x}{2}\right) - 16 \sin\left(\frac{x}{2}\right) + C$$

2x	2 sin(x/2)
2	-4 cos(x/2)
0	-8 sin(x/2)

Solve the initial value problem using tabular integration

Solve the initial value problem

$$11. \left\{ \frac{dy}{dx} = \begin{array}{l} (x+2) \sin x \\ \hline 1 \\ \hline 0 \end{array} \right. \quad y = 2 \text{ and } x = 0$$

$\swarrow$   $-\cos x$   
 $\swarrow$   $-\sin x$

$$y = -(x+2) \cos x + \sin x + C$$

$$2 = -2 + C$$

$$4 = C$$

$$y = -(x+2) \cos x + \sin x + 4$$

$$16. \left\{ \frac{dy}{dx} = \begin{array}{l} 2x\sqrt{x+2} \\ \hline 2 \\ \hline 0 \end{array} \right. \quad y(-1) = 0$$

$$y = \int \begin{array}{l} 2x(x+2)^{1/2} \\ \hline 2 \\ \hline 0 \end{array} dx$$

$\swarrow$   $\frac{2}{3}(x+2)^{3/2}$   
 $\swarrow$   $\frac{4}{15}(x+2)^{5/2}$

$$y = \frac{4x}{3} (x+2)^{3/2} - \frac{8}{15} (x+2)^{5/2} + C$$

$$0 = -\frac{4(1)}{3(5)} - \frac{8}{15} + C$$

$$y = \frac{4x}{3} (x+2)^{3/2} - \frac{8}{15} (x+2)^{5/2} + \frac{28}{15}$$

12 | Page  $C = \frac{28}{15}$

$$0 = -\frac{20}{15} - \frac{8}{15} + C$$

Use tabular integration to integrate the following

$$10. \int x^2 \ln x dx$$

$2x$	$x \ln x - x$
$2$	$?$
$0$	$0$

$\ln x$	$x^2 dx$
$\frac{1}{x}$	$\frac{1}{3} x^3$

$$\int x^2 \ln x dx = \frac{1}{3} x^3 \ln x - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \frac{1}{3} x^3 \ln x - \frac{1}{3} \int x^2 dx$$

$$= \frac{1}{3} x^3 \ln x - \frac{1}{3} \cdot \frac{1}{3} x^3 + C$$

Use ultra violet minus super vdu to integrate the following

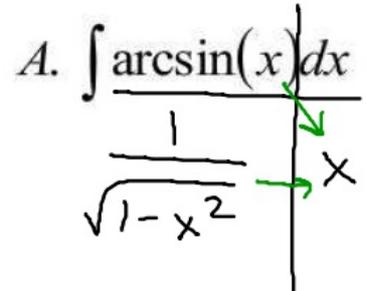
$$10. \int x^2 \ln x dx$$

$$\int \ln x dx = x \ln x - \int 1 dx$$

$\frac{1}{x}$	$x$
---------------	-----

$$= x \ln x - x + C$$

Use tabular integration to integrate the following

$$A. \int \frac{\arcsin(x)}{\sqrt{1-x^2}} dx = x \arcsin x - \int x(1-x^2)^{-1/2}$$

$$= x \arcsin x + (1-x^2)^{1/2} + C$$

$$19. \int e^x \cos(2x) dx$$